Uncertainty and Aeroelasticity: ONERA's Experience Review

Fabrice POIRION
ONERA/DDSS
BP 72, 92322 Chatillon
FRANCE

poirion@onera.fr

ABSTRACT

Uncertainty effects on the stability of fluid--structure coupled systems have been investigated at ONERA for more than a decade. Stability of the coupled system aircraft/aerodynamics is a key issue in airplane manufacturing. During the design stage of an airplane, many structural parameters are not clearly fixed nor known, but nevertheless the final project must comply to the various international certification regulations. One popular approach to deal with this problem is to model the parametric uncertainties through random variables. Then one has to study the stability of a random parameter dynamical system. After presenting the various steps developed at ONERA in order to construct an effective numerical procedure which can be utilized together with standard structural and aerodynamic codes by manufacturers, we focus on various models for introducing uncertainty in an elementary stiffness matrix. It is shown that a chaos polynomial representation of the matrix whose coefficients are estimated through Monte Carlo simulation yields a better approximation for the matrix and its eigen values than a Taylor expansion which is currently used in the literature. The method is then applied to an aircraft model. .Besides conception uncertainties there exist other source of uncertainties appearing in an aircraft: we give two illustrations related to aerodynamic and aeroservoelastic applications.

1.0 INTRODUCTION

Stability of the coupled system aerodynamics/aircraft is a key issue in aircraft manufacturing. During the design process of a new airplane, many structural parameters are not yet frozen and designers must show that, within the entire range of possible parameter values, the airplane will not encounter unstable motions in the flight domain. The parameter uncertainties are modelled as random variables whose distributions are chosen according to available manufacturer knowledge: for instance, the Young modulus of certain carbon materials can vary quite significantly from a sample to another, and due to the limited set of data available, the dispersion of this parameter can be important, leading to a maximum uncertainty up to 100\%. The critical or flutter speed is defined as the lowest aircraft speed, if any, for which the plane becomes unstable. When random parameters are introduced in the model this quantity becomes a random variable which has to be characterized: mean value, standard deviation, probability distribution … Moreover the notion of stability needs to be replaced with the notion of "flutter probability". From perturbation techniques to Monte Carlo simulation (MCS) methods, there exist several methods [5,7,8] which can be used to characterize the probabilistic flutter speed, depending on the numerical effort that manufacturers can afford, the least expensive one giving of course the crudest results. Those methods have been developed and tested at ONERA on realistic models of airplane. This paper will go over their descriptions, highlighting the pros and cons of each one, keeping in mind the manufacturer requirement of the computational efficiency and accuracy. Finally a method based on MCS and basis reduction will be proposed. It is based on a polynomial chaos expansion of the random matrices. An illustration on a 20000

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DOF airplane model is then illustrated. Here, it is the elasticity values of the wing structure which are considered uncertain. Taylor and polynomial chaos approaches are used to estimate second order statistics of the generalized stiffness matrix (four elastic modes are considered). Results are compared and discussed. The model is then introduced in the flutter equation. Another point which is considered is the problem of aerodynamic uncertainties: where do they have to be introduced? Considering a 2 degree of freedom airfoil, we study the effect of shape uncertainties on the stability diagram. We end with the study of uncertain time delay effect in an aeroservoelastic problem.

2.0 PARAMETRIC UNCERTAINTY PROPAGATION

2.1 Aeroelastic Model

The flutter equation depicts the coupling between the structure (airplane) and the fluid (aerodynamic forces). This coupling can induce either sustained bounded oscillations (limit cycles) which contribute to overall aircraft fatigue, or explosive and destructive oscillations (divergent flutter). Formulating the problem mathematically requires writing the fluid equations, the structural equations and the interactions at the different interfaces.

The coupling aerodynamic forces A between fluid and structure are assumed to be known and are considered as particular external forces. Using a finite element model of the airplane, the flutter equation can be written:

$$M\ddot{u}(x,t)+Ku(x,t)=A(u,\dot{u},\ddot{u},t) \tag{1}$$

Due to the dimension of the FEM (up to 10^6 DOF), the equation is written using the modal basis of the structure. The motion of a point x of structure is expressed as a finite linear combination of eigen modes, retaining the N first low-frequency modes:

$$u(x,t)=\sum_{j=1}^{N} q_j(t)\Phi^j(x), \tag{2}$$

where $\Phi^j(x)\, j=1,N$, is a finite family of elementary displacement vectors which satisfy the following properties:

$$-\omega_j^2 M\Phi^j + K\Phi^j = 0, \tag{3}$$

$$< M\Phi^j,\Phi^\ell >= \mu_j\delta_{j,\ell} \quad < K\Phi^j,\Phi^\ell >= \mu_j\omega_j^2\delta_{j,\ell}, \tag{4}$$

where M and K denote the respective associated mass and stiffness linear operators. In the frequency domain, the flutter equation can be written in the modal basis:

$$(-\omega^2\mu+\kappa)\tilde{q}(\omega)=\alpha(i\omega,V)\tilde{q}(\omega) \tag{5}$$

where μ and κ are the $N\times N$ diagonal generalized mass and stiffness matrices, $\tilde{q}(\omega)=(\tilde{q}_1(\omega),\ldots,\tilde{q}_N(\omega))$ is the vector whose coordinates are the Fourier transform of the generalized coordinates $q_j(t), j=1,N$, $\alpha(i\omega,V)$ is the $N\times N$ complex generalized aerodynamic coefficient matrix due to the aircraft motion, for a given speed V. Structural damping, which is usually not well known, is not introduced in what follows.

The generalized aerodynamic forces depend on any structural uncertainty introduced in the dynamic model. Let $\varepsilon=(\varepsilon_1,\ldots,\varepsilon_N)$ represents N uncertain independent parameters. Equation (5) has to be replaced with:

$$(p(\varepsilon)^2\mu(\varepsilon)+\kappa(\varepsilon)-\alpha(p(\varepsilon),V))X(\varepsilon)=0 \tag{6}$$

$p(\varepsilon)=\xi(\varepsilon)+i\omega(\varepsilon)\in\mathbb{C}$, $X(\varepsilon)\in\mathbb{R}^N$ where the new generalized matrices depend on the uncertain parameters. N is the number of modes retained ($N\le 50$). The aircraft stability at a fixed speed V is driven by the sign of $\xi(\varepsilon) = \mathrm{Real}(p(\varepsilon))$ for any solution $p(\varepsilon)$. The uncertain parameters are idealized as random variables whose probability distribution is chosen for instance following the maximum entropy principle [6].

2.2 Monte Carlo Simulation Approach

The prodigious development of digital computer has established the Monte Carlo simulation method as the most reliable, general purpose approach in stochastic mechanics. Accurate solutions can be obtained for any problem whose deterministic solution (analytical or numerical) is known: a large number of realizations of the random parameters are generated based on their known statistical description (this generation can be done directly in the FEM data file) and to each realization a result (the solution of the flutter equation) is calculated as in deterministic analysis. Finally the results are examined by statistical methods. The method is however time consuming since the computation of each realization requires a full finite element analysis (in order to construct the updated modal basis) together with a full aerodynamic computation.

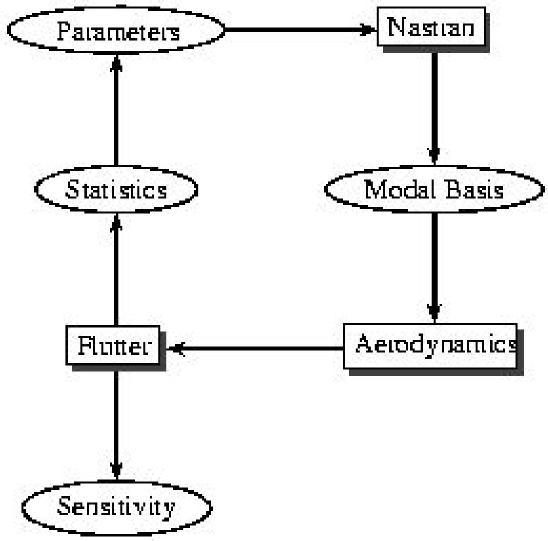


Figure 1: Monte Carlo procedure

2.3 Perturbation Approach

Each solution of the flutter equation is perturbed about the mean value of the uncertain parameters through a second-order Taylor expansion around the mean value of the uncertain parameters obtained for $\varepsilon = 0$:

$$p(\varepsilon) \approx p(0) + \sum_{j=1}^{N} \frac{\partial p}{\partial \varepsilon_j}(0)\,\varepsilon_j + \sum_{j,l=1}^{N} \frac{\partial^2 p}{\partial \varepsilon_j \partial \varepsilon_l}(0)\,\varepsilon_j \varepsilon_l \tag{7}$$

The main difficulty is the determination of the various derivatives appearing in the above equation. The first order derivatives can be obtained directly through NASTRAN aeroelastic package. Algebraic expressions for the second order derivatives can be written but they lead to intractable calculations and they have to be calculated numerically. The flutter probability is then defined for a given speed by:

$$\mathrm{P}\{\mathrm{Real}(p) > 0\} = \int_{N} {}_{\{\mathrm{Real}(p)\geq 0\}}(\varepsilon) P_{\varepsilon}(d\varepsilon) \tag{8}$$

and can be calculated analytically or numerically using the above Taylor expansion. However, despite its numerical efficiency, it does not give accurate enough results and moreover is mathematically valid only for small uncertainties. In [8] is highlighted the discrepancy of the standard deviation of the flutter solution real part when this method is used compared to the MCS reference solution after having introduced thickness uncertainty in the wing panels. Moreover it is shown that a second order expansion does not improve the accuracy compared to a first order expansion for this particular example.

2.4 Projected Monte Carlo Approach

In order to suppress the necessity of computing the new modal basis at each realization of the random parameters in the MCS method, a simple idea is to represent all the generated mechanical system using an unique basis: the mean (or nominal) modal basis $\Phi(0)=(\Phi^j(0))_j$ which is constructed from the zero uncertainty structural model. The uncertainty effects will then appear only through the generalized matrices $\mu(\varepsilon)=\Phi(0)^T M(\varepsilon)\Phi(0)$ and $\kappa(\varepsilon)=\Phi(0)^T K(\varepsilon)\Phi(0)$ expressions in equation (5). Of course one needs to describe the dependency of the applications $\varepsilon \mapsto \mu(\varepsilon)$ and $\varepsilon \mapsto \kappa(\varepsilon)$, which is achieved using a first order Taylor expansion :

$$\mu(\varepsilon) \approx \mu(0)+\sum_{i=1}^{N} \varepsilon_i \Phi(0)^T \partial_{\varepsilon_i} M(\varepsilon)|_{\varepsilon=0} \Phi(0) \tag{9}$$

$$\kappa(\varepsilon) \approx \kappa(0)+\sum_{i=1}^{N} \varepsilon_i \Phi(0)^T \partial_{\varepsilon_i} K(\varepsilon)|_{\varepsilon=0} \Phi(0) \tag{10}$$

The sensitivity matrices $\partial_{\varepsilon_i} M(\varepsilon)$ and $\partial_{\varepsilon_i} K(\varepsilon)$ can be evaluated relatively easily using specific routines of FEM codes such as for instance NASTRAN. The aerodynamic forces depend only on the wing geometry and therefore the generalized aerodynamic matrix α has, in this approach, to be constructed only once (if the aerodynamic surfaces are fixed). This approach is numerically very efficient since one has to solve a low dimension problem described by equation (6) using a MCS method. Nonetheless it suffers two major weakness: the first one is that expansions (9) and (10) are meaningful only for small uncertainties. The second one is that the mean modal basis may not be representative enough to allow the representation of all the randomly generated systems mode shapes [8] .

2.5 Extended Modal Basis

In order to work out the second weakness of the projected MCS method, a natural approach is to consider a richer or more representative basis on which the flutter equation will be projected. But due to the specific mechanical interpretation of the modal basis and since aerodynamics codes are developed in this context, it is convenient to keep the mean modal basis as the core of the new basis and to complete it with a set of linearly independent vectors. The choice for those vectors are arbitrary but a judicious one is to consider the derivative vectors $(\frac{\partial \Phi}{\partial \varepsilon_k})|_0$ which characterize the sensitivity of the mode shapes respectively to each uncertain parameter. Moreover the derivative of the modes can be directly obtained in FEM codes. The extended modal basis approach involves therefore the projection of the flutter equation on the basis $\Theta=[\Phi(0),\Psi]$ where $\Phi(0)$ is the mean modal basis and Ψ is a set of independent vectors extracted from the family $(\frac{\partial \Phi}{\partial \varepsilon_k})|_0$. Once the new generalized matrices are constructed, the simulation procedure is exactly the same as in the projected MCS method. Moreover, when projecting the new equation in the modal basis of the reduced mechanical system $\{\Theta^t M(0)\Theta, \Theta^t K(0)\Theta\}$, it is possible to work again within the classical framework of physical structural modes.

2.6 Hermite Polynomial Chaos Projection

Our goal now, is to find a better probabilistic description or model of the random generalized matrices which could be used for the projected MCS method. Hermite polynomial chaos decomposition is a technique commonly used to express a random function in terms of Gaussian variables. Let $\xi = (\xi_j)_{j\in}$ a set of orthonormal Gaussian random variables defined on a probability space (Ω, F, P). Then any square integrable random function $\Omega \ni \omega \mapsto f(\omega)$ can be approximated in the Hilbert space $L^2(\Omega)$ by :

$$f(\omega) = f_0 + \sum_i f_i \Gamma_1(\xi_i) + \sum_{i,j} f_{i,j} \Gamma_2(\xi_i, \xi_j) + ... \tag{11}$$

where Γ_I are Hermite polynomials: $\Gamma_1(\xi_i) = \xi_i$, $\Gamma_2(\xi_i, \xi_j) = \xi_i \xi_j - \delta_{i,j}, ...$ and where $f_{\mathbf{k}} = E(f \times \Gamma_p(\xi_{\mathbf{k}}))$ for any multi-index $\mathbf{k} = (k_1, ..., k_p)$ and E denotes the mathematical expectation operator. Applying this expansion method to our problem, the Taylor's expansion (9) and (10) have to be replaced with truncated expansions which can formally be written:

$$\mu(\varepsilon) \approx \mu_0 + \sum_{I=1}^{Q} \mu_I \Gamma_I(\xi_Q) \kappa(\varepsilon) \approx \kappa_0 + \sum_{I=1}^{Q} \kappa_I \Gamma_I(\xi_Q) \tag{12}$$

where ξ_Q denotes a finite family of ξ. The difficulty of this approach lies in the determination of the expansion coefficients $\mu_I = E(\mu \times \Gamma_I(\xi))$ and $\kappa_I = E(\kappa \times \Gamma_I(\xi))$. They are given mathematically by integrals involving the probability distributions of the random vectors $[\mu(\varepsilon), \xi_Q]$ and $[\kappa(\varepsilon), \xi_Q]$ which are unknown in the general case. But they can also be numerically estimated using a MCS method which will generate samples of the random vector $[\varepsilon, \xi_Q]$. Unfortunately the distribution of this vector is not known either. One solution is to express each random parameter ε_i itself as a function of Gaussian variables. This can be done for a very large class of well behaved random variables Y, using relation:

$$Y = F_Y^{-1}[F_G(G)] \tag{13}$$

where G is a standard normal random variable, F_G and F_Y the cumulative distribution function of G and Y respectively. Other direct transformations can be written. For instance, if U has a $[0,1]$ uniform distribution then it can be expressed in terms of two orthonormal Gaussian variables:

$$U = \exp \frac{-1}{2}(\xi_1^2 + \xi_2^2) \tag{14}$$

The polynomial chaos expansion of U can be readily obtained using the exponential function series:

$$U = \sum_{n=0}^{\infty} (\frac{-1}{2})^n \frac{(\xi_1^2 + \xi_2^2)^n}{n!}. \tag{15}$$

This last example is interesting for our particular problem since uncertain parameters are usually modeled through uniform distributed random variables. In that case each matrix $\mu(\varepsilon)$ and $\kappa(\varepsilon)$ can be expressed as a polynomial chaos expansion involving exactly $2 \times N$ Gaussian orthonormal random variables. And now the coefficients of expression (12) can be estimated very easily using simulations of the independent Gaussian variables $\xi = (\xi_1, ..., \xi_M)$:

$$\kappa_I = E(\kappa \times \Gamma_I(\xi)) \approx 1/N \sum_{\ell=1}^{N} \kappa(\xi^\ell) \times \Gamma_I(\xi^\ell) \tag{16}$$

where ξ^ℓ denotes a generated value of Gaussian vector .

2.7 Illustration

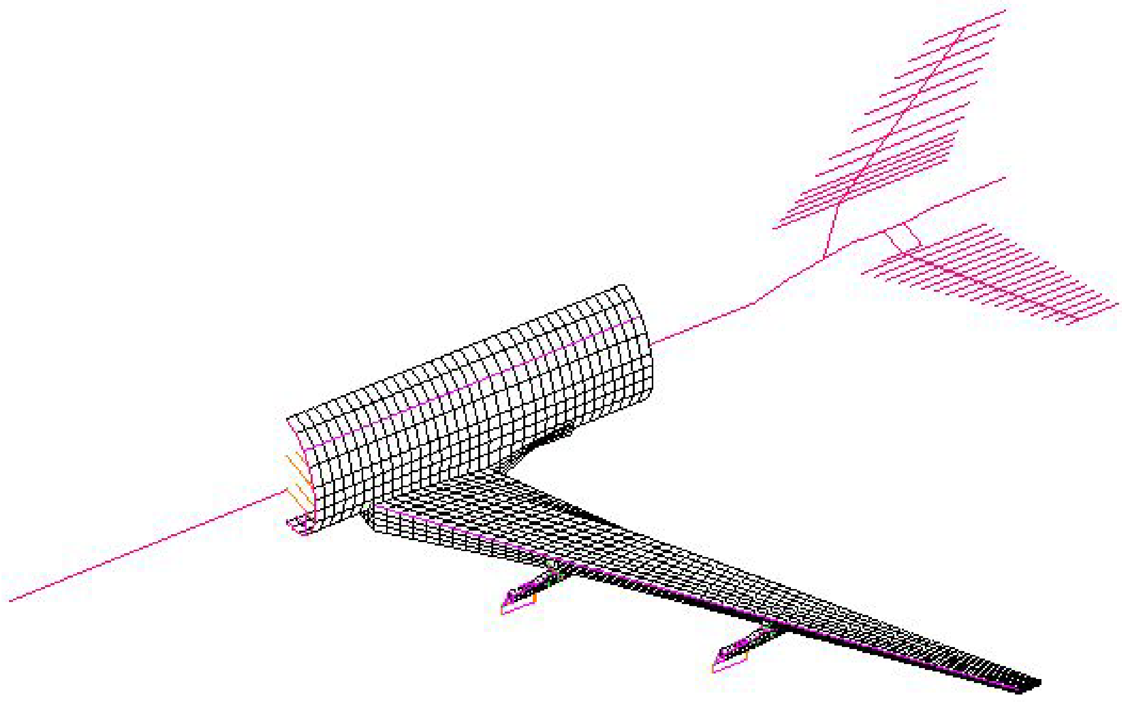


Figure 2: 20000 DOF airplane finite element model - 4 uncertain material elasticity parameters

We consider a 20000 DOF airplane model (Fig. 2) in which the elasticity of four wing materials is considered uncertain and modeled as independent random variables (only the red part of the airplane model is described with a fixed elasticity value): $\varepsilon_i = \varepsilon_{i,0}(1+\alpha u_i)$, $i = 1,4$, where u_i are $[0,1]$ uniform random variables and α_i are uncertainty coefficients. In this model, the lowest elasticity values are fixed. In a first stage we look at the generalized stiffness matrix statistics $\kappa(\varepsilon)$ when four elastic modes are used for the projection onto the truncated modal basis. The reference solution is constructed using the complete MCS method described in section 3. Then, using sensitivity matrices obtained in NASTRAN, a first order Taylor expansion is constructed for matrix κ, (relation 10). Finally a second order polynomial chaos expansion (relation 12) is constructed, its coefficient being estimated using a MCS method, (relation 16). This last model yields a diagonal matrix when the linear perturbation model gives a non diagonal one. In the following applications, comparisons between the eigen values of the two models will be given. The reference solution is estimated using 900 samples when a relative uncertainty of 40 % is introduced in the model, $\alpha_i = .4$. The polynomial chaos coefficients are estimated using 200 samples. The two models are then used in order to generate 1000 samples of the random generalized stiffness matrix. The mean value and standard deviation of each of the four eigen values are finally estimated from those last samples. Table 4 gives the relative error (relatively to the reference solution) for those estimates. Those last results emphasize the influence of the estimate accuracy of the chaos expansion on the model overall accuracy. This can bring a limitation to the method if a large number of samples are needed. In that last case it is better to use the standard MCS method.

$$\begin{array}{cc} \text{chaos expansion} & \text{Taylor expansion} \\ \begin{pmatrix} 1.4 \\ 2.4 \\ 3.2 \\ 3.6 \end{pmatrix} & \begin{pmatrix} 7 \\ 10 \\ 12.6 \\ 13 \end{pmatrix} \end{array}$$

Table 4. Eigen value mean value relative error

$$\begin{array}{cc} \text{chaos expansion} & \text{Taylor expansion} \\ \begin{pmatrix} 25 \\ 21 \\ 24 \\ 9 \end{pmatrix} & \begin{pmatrix} 12 \\ 14 \\ 23 \\ 44 \end{pmatrix} \end{array}$$

Table 5. Eigen value standard deviation relative error

One can all ready notice the good behavior of the polynomial chaos model which gives a better estimate for the eigen value mean values. However the errors on the standard deviation are of the same order than those obtained using the Taylor's model. The standard deviation errors decrease as the number of samples used increase, as it is illustrated on figure 3 where the error (mean value and standard deviation) evolution for each eigen value versus the number of samples is plotted, together with the errors corresponding to the Taylor's approach. The next step is to introduce the polynomial chaos expansion model of the stiffness matrix (relation (12)) into the flutter equation 5. A reference solution is constructed using a complete MCS method as described in section 3. Nine hundred samples (necessitating around two dozen hours of calculation on a workstation) are used in order to obtain the "mean flutter diagram" given in figure 4. The plain lines represent the mean values while the dotted lines represent the quantity $E(p) \pm \sigma_p$ for each solution p of the flutter equation. The two models, Taylor and polynomial expansion, are then compared in respectively figure 5 and figure 6. One can check that the linear perturbation approach (Taylor) does not give the general behavior of the reference solution, contrary to the second model. The chaos coefficients used in the later were estimated using 400 samples, and the statistics appearing on the figure were estimated over 1000 samples. Compared to the reference case, it yields more scattered results, but a good estimation of the mean values. The deviation tends to increase when fewer samples are used in order to estimate those coefficients. Finally, if we go back to the reference solution obtained using a complete MCS method, it appears that the second order statistics converge rapidly respectively to the number of samples: in figure 8, we have plotted for each mode the evolution of its damping second order moment for each speed value introduced in the flutter calculation with respect to the number of simulation. One can check that convergence is obtained after 400 simulations. This last figure is to be compared to the number of samples used to estimate the polynomial chaos coefficient.

2.8 Remarks

Because flutter calculations involve highly nonlinear equations, MCS methods are the more appropriate ones for introducing random uncertain parameters in the model and evaluating their impacts on the stability of the coupled system aircraft/aerodynamics. Even if the complete MCS method will give the more accurate results, nevertheless reduced methods are required today in order to address the computational cost due to the high dimension of the finite element models used by manufacturers. A feasible and effective approach has been presented, based on the construction of an unique representative basis on which the flutter equation is projected. Moreover, in order to obtain a more accurate representation of the uncertainty propagation in the various structural matrices, the use of Hermite polynomial chaos has been suggested, instead of the classical first order perturbation technique. A simple example has shown the benefit of this approach, together with some of its difficulties. As far as this study in concerned, it appears that a second order polynomial chaos expansion is sufficient to obtain satisfactory results. The second application on a realistic case has shown the feasibility of the polynomial chaos approach for modeling random structural matrices in real life engineering structures, but has also highlighted the importance of using accurate coefficients in the expansion. Since those coefficients are obtained numerically (either using adapted numerical integration methods when few uncertain parameters are involved or using MCS as soon as more than four parameters are introduced) for nonlinear problems, particular efforts are still needed to improve their accuracy and limit their computational cost. Finally the question arises whether such simplified approaches should be used, at least for middle size (several dozens of thousand degrees of freedom) models are involved, since MCS methods appear to converge rapidly (at least for the flutter problem and since the computational power of computers increases continuously.

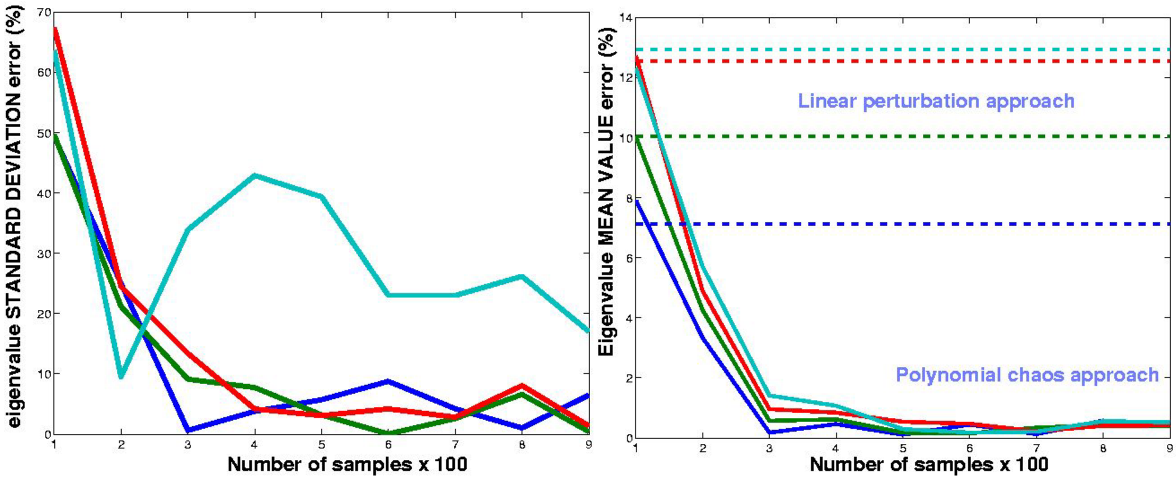
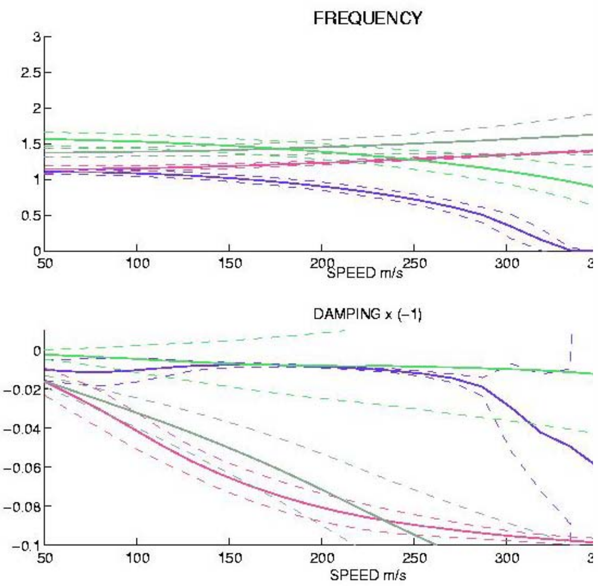


Figure 3: Effect of the number of samples on the estimation errors



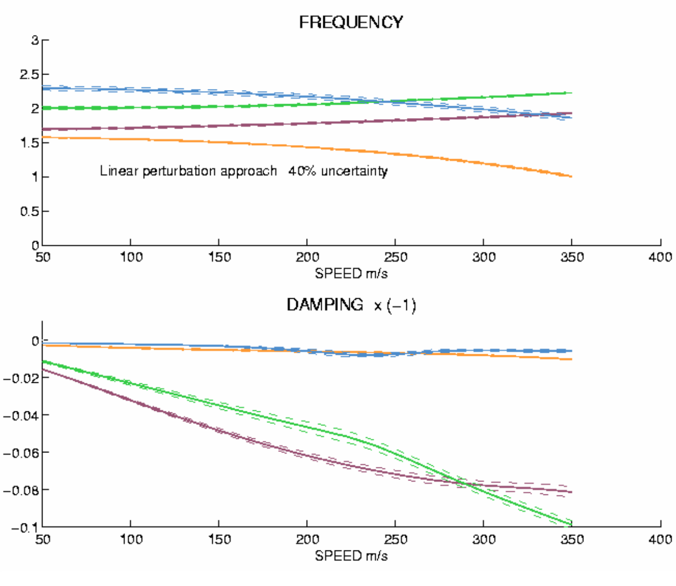


Figure 4: Full MCS method - mean and std **Figure 5: Taylor approach - mean and std**

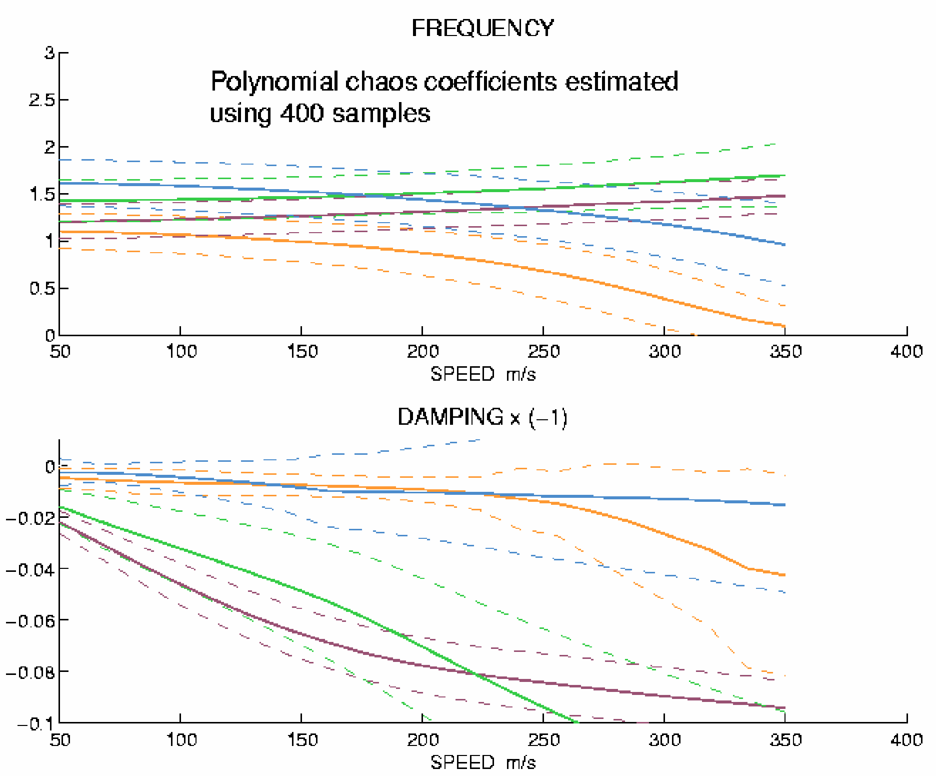


Figure 6: polynomial chaos - mean and std

3.0 AERODYNAMIC UNCERTAINTY

Contrary to structural uncertainties, it not so easy to localize relevant aerodynamic parameters where uncertainty can be introduced. Should they model the discrepancy between different aerodynamic codes, the effect of simplification assumptions or more physical aspects such as the position of the shock on an airfoil or the coefficient of pressure? Moreover the difficulty comes from the fact that solutions of the flutter equation do not depend in a functional way on those aerodynamic coefficients since aero dynamical forces are obtained by numerical integration of complex equations such as Euler or Navier Stokes equation. This problem will be clarified by considering the simple configuration of a two-dimensional airfoil. In that case, the sole aerodynamic parameter which has to be considered is the airfoil geometry. After having defined the flutter problem considered, a random field modeling airfoil geometric deformations will be constructed, then a Monte Carlo simulation method will be described and applied on a given airfoil. Results will show the effects of geometric defects on the coefficient of pressure and on the flutter equation solutions.

3.1 Random Deformation of a Two-Dimensional Airfoil

The goal of this section is to define random deformation of an airfoil profile which could depict for example processing imperfections yielding local modifications of the airfoil slope with humps and hollows. The common way to describe a profile P is to define a finite number of points $\{M_1, M_2, ..., M_p\}$ laying on it. Two different approaches can be utilized in order to generate random fluctuation of a profile geometry. The first one is to change randomly and independently the position of each point describing the profile. This approach will introduce very strong discontinuities in the profile slope, which could bring serious convergence problems in the aerodynamic codes. Moreover it is dependent on the number of points used to describe the profile. The second approach is to consider a continuous deformation field d; $M \mapsto d(M), M \in P$. The deformation at a point $M \in P$ is equivalent to a modification of the local curvature at that point and therefore will be defined by a normal vector at point M.

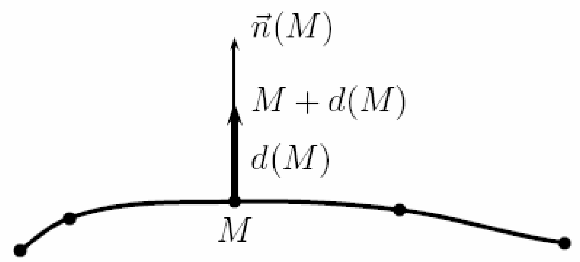


Figure7. Deformation field

Definition Let P a profile defined by a $C^k, k \geq 2$ curve parameterized by arc-length. The deformation random field d of profile P is modeled by a second-order, zero-mean, mean-square continuous random field, indexed on the curve P with real values.
Let P be a given profile. Its random deformation is defined by the following application

$$M \mapsto M + d(M)\vec{n}(M); M \in P$$

where $\vec{n}(M)$ is the normal vector at point M on the profile.
Let $R_d(M, M') = E(d(M)d(M'))$ denote the random deformation autocorrelation function.
The deformation *d(M)* is numerically simulated using its Karhunen Loeve Expansion:

$$\forall M \in P, d(M) = \sum_{\alpha \geq 1} \sqrt{\lambda_\alpha} \xi_\alpha \phi_\alpha(M)$$

where $\int_P R_d(M, M')\phi(M')dM' = \lambda\phi(M)$ and where $\xi_1, \xi_2, ..., \xi_\alpha, ...$ are uncorrelated random variables given by

$$\xi_\alpha = \frac{1}{\sqrt{\lambda_\alpha}} \int_P < d(M), \phi_\alpha(M) > dM.$$

3.2 Illustration

We consider the NACA64A010 profile defined through $p = 132$ points M_k, with the parameter x of the curve taking its values in $I = [0,1]$.
The autocorrelation function of the deformation random field $d(M(x))$ is chosen as:

$$R_d(x, x') = \sigma^2 \exp(-C \,|\, x' - x \,|), \tag{16}$$

with a standard deviation $\sigma = .1$. The correlation length depends on the value of parameter C: the greater it will be, the more independent will be the deformation between two points on the profile. The following two figures show an example of the deformation field effect when $C = 1$ and $C = 100$.

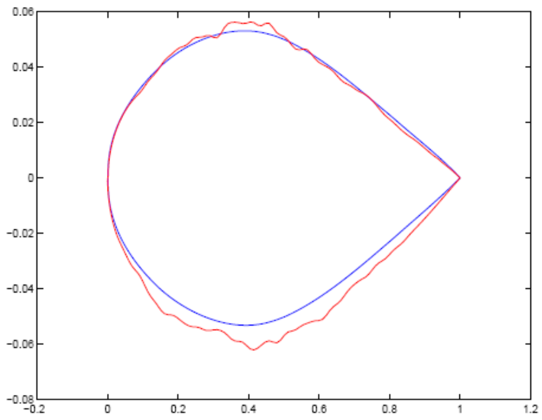


Figure 8. Deformation field for $C = 1$

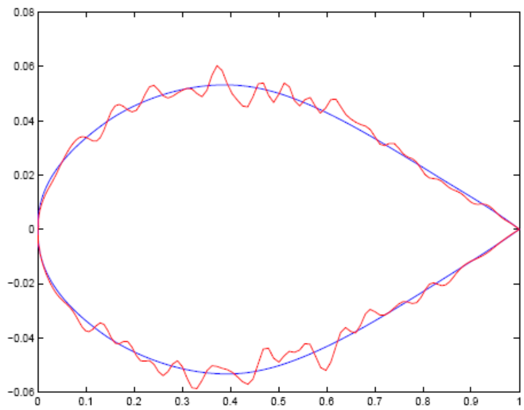


Figure 9. Deformation field for $C = 100$

A Monte Carlo procedure is utilized in order to perform an aerodynamic calculation (Euler equation) for each generated deformed profile. The correlation constant was fixed to $C = 4$, and 1000 simulations were performed.

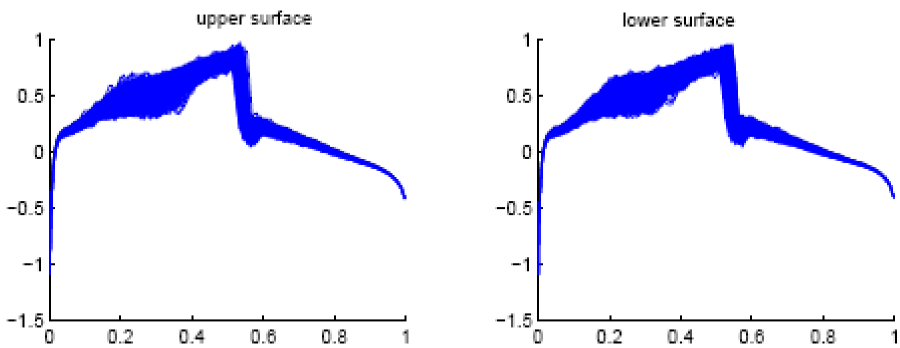


Figure 10. Pressure coefficient - 1000 simulations

Figure 9 represents the scattering of the different pressure coefficient evolution curves versus the position on the airfoil for the different shape perturbed airfoil. The different curves are rather jagged, which results from the fact the pressure coefficient is proportional to the airfoil slope. However the shock position does not change dramatically.

The flutter speed (or critical speed) where the coupled system becomes unstable, is a random variable which histogram is given on figure 11.

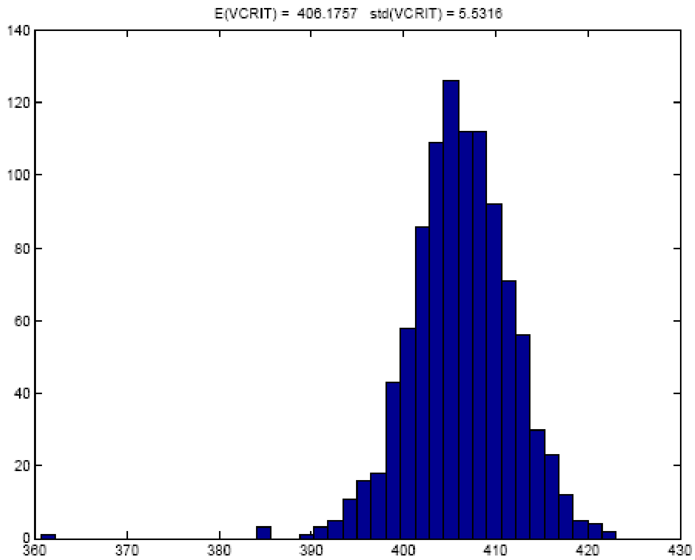


Figure 11. Critical speed histogram

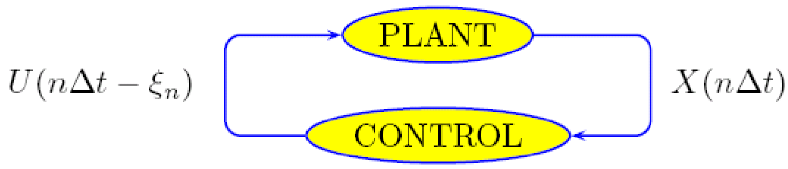
4.0 UNCERTAIN TIME DELAY IN CONTROLED SYSTEMS

The use of active control technology significantly enhances aircraft stability and response, at reduced structural weight. In particular, future high capacity aircraft will be characterised by overlapping of rigid and flexible modes which will induce severe interactions between the structure and controls. The effects of fly-by-wire controls on flutter must therefore be examined carefully.

A particular aspect of this problem is the influence of time delays which causes unsynchronized application of the feedback control force and consequently can render the control ineffective and furthermore may destabilise the system. Fly-by-wire controls give rise to many sources of time delays which occur for instance during data acquisition, control law computation time, data transfer, etc. The numerical values of those delays can be determined once the sampling rates and number of computers are known.

When redundant calculations are performed as it is the case for modern transport aircraft for which safety policy requires using several different data processor channels to calculate the same quantities in order to cater for failure of one of the computers, the time delay values vary during flight. Whereas a known and constant time delay can be compensated for --- the study of delayed systems has received considerable attention in the control theory community} --- this is not true for unknown time-varying ones. Moreover the classical frequency domain approach using the Laplace transform can no longer be used and a direct analysis of the time domain response of the system has to be made in order to study the stability.

The purpose of this section is to show that in this case flutter can be analysed by borrowing a numerical stochastic analysis tool developed for studying the stability of stochastic dynamical systems. This approach is based on the fact that a linear differential equation with random time delay can be seen in a certain functional space as an infinite-dimensional linear differential equation with stochastic multiplicative noise. Once discretized, this equation is approximated by a finite-dimensional linear system with a random matrix which describes the motion of a discrete dynamical system with multiplicative noise. The stability of its solution can then be studied using the stochastic Lyapounov exponent method. This method does not yield the usual flutter diagram showing the variation of the damping and frequency of each mode but instead gives a stability domain in terms of delay values and aircraft speeds.



In practice the closed loop control law uses the value X_n of the observation sampled at time $t_n = n\Delta t$ to compute the value U_n of the control at the same time.
But due to the time required to perform the various calculations and samplings there exists a delay ξ between the moment when the value of the control should be available and the moment when it is effectively available. For complex controlled systems such as the ones encountered on modern fly-by-wire aircraft, this delay is unknown and depends moreover on the time t_n when the calculation occurs. Therefore the delay at time t_n can be seen as a random variable $\tau_n(a); a \in A$ defined on a given probability space $(A, \mathcal{T}, P)$. Letting the time step to approach 0, the delay can be modelled as a real-valued stochastic process $\tau(t,a)$. The aeroservoelastic system equation can be written as a delayed differential equation with random delays.

$$\dot{X}(t) = AX(t) + BX(t-\tau)$$

It can be shown [4,9,12] that there exists a linear operator $L \in L(C,)$ such that any solution of the delayed equation $\dot{x}(t) = Ax(t) + Bx(t-\tau)$ is also a solution of the functional delayed equation

$$\frac{d}{dt}x(t) = L(\tilde{x}_t).$$

Operator L is given by :

$$L : C \to \phi \mapsto \int_{-\tau}^{0} [d_\theta \eta(t,\theta)]\phi(\theta), \text{with}$$

$$\eta(t,\theta) = \begin{cases} -A(t) - B(t) & \theta - \tau, \\ -A(t) & -\tau < \theta < 0 \\ 0 & \theta = 0 \end{cases}$$

$$\text{where} \quad \tilde{x}_t(\theta) \overset{\text{def}}{=} x(t+\theta); \quad -\tau \le \theta \le 0$$

This formalism can be extended to the case of a time varying random delay $\xi(t,a)$. Finally, a linear random-delayed differential equation can be written as a linear stochastic system with multiplicative noise:

$$\frac{d}{dt}x(t) = L(\xi(t,a), \tilde{x}_t).$$

When working in the time domain, the flutter equation reduces to the stability study of a one-parameter (speed or pressure) dynamical system describing the aeroservoelastic equation. When the system is also linear, stability results are obtained immediately by looking at the characteristic equation roots. In the case of a random delay, stochastic tools must be used. Let a linear dynamical system be described by the following equation,

$$\dot{x}(t) = Ax(t), x(0) = x_0, \quad x(t), x_0 \in \mathbb{R}^n.$$

The stability property characterizes the proximity of two solutions $x(t, x_0)$ with close initial values. In the same way, the asymptotic stability characterizes the long term behavior of the solution: the zero solution ($x(0) = 0$) of equation $\dot{x}(t) = Ax(t)$ is said to be asymptotically stable if for all small initial values

$||x_0|| \leq \eta$, $||x(t,x_0)|| \to 0$, when $t \to +\infty$. The following result characterizes such systems: the zero solution is asymptotically stable if and only if the eigen values of matrix A have negative real parts. If no delays occur in the various control systems when using a state space model for the unsteady aerodynamic forces, the aeroservoelastic system motion is driven by a linear differential equation, and its stability is obtained trivially using this last result. The notion of asymptotic stability can be extended for delayed differential equations. Concerning stochastic differential equations, the almost sure stability notion is used: the zero solution of a linear differential equation with multiplicative noise is called almost surely (a.s.) asymptotically stable if for almost all fixed $a \in A$ the solution $t \mapsto x(t,a)$ is asymptotically stable.

The next result generalizes the classical result: a linear stochastic system $\dot{x}(t) = A(\xi(t,a))x(t))$, $x(0) = x_0$ is a.s. asymptotically stable if and only if

$$\mathrm{Prob}(\lim_{t\to\infty} ||x(t,x_0,a)|| = 0) = 1$$

for all initial value $x_0 \in \mathbb{R}^n$.

The Lyapounov exponent gives an effective tool for studying stochastic system stability. Definition The Lyapounov exponent of a solution $x(t,x_0)$ of a linear stochastic differential equation is defined by:

$$\lambda(x_0,a) = \lim_{t\to\infty} \frac{1}{t} \log ||x(t,x_0,a)|| \, a \in A$$

It is a random variable which depends on the initial value x_0 and may be interpreted as the exponential growth rate of the solution. The last result characterizes asymptotically stable solutions using Lyapounov exponents. a linear stochastic system is a.s. asymptotically stable if and only if

$$\mathrm{Prob}(\max_{x_0 \neq 0} \lambda(x_0,a) < 0) = 1$$

Additional smoothness assumptions must be made for the random noise $\xi(t,a)$ to have access to effective tools. In this paper, the noise is modeled as the solution of an Ito stochastic differential equation:

$$d\xi(t) = a\,\xi(t)dt + b\,dW(t)\xi(t) \in \mathbb{R}.$$

In this case, it can be shown that the random variable $\lambda(x_0,a)$ has only a finite number of values

$$\lambda(x_0,a) \in \{\lambda_q < \lambda_{q-1} < \ldots < \lambda_1 = \lambda_{\max}\}.$$

The system stability then depends on the sign of the greatest Lyapounov exponent $\lambda_{\max}$.

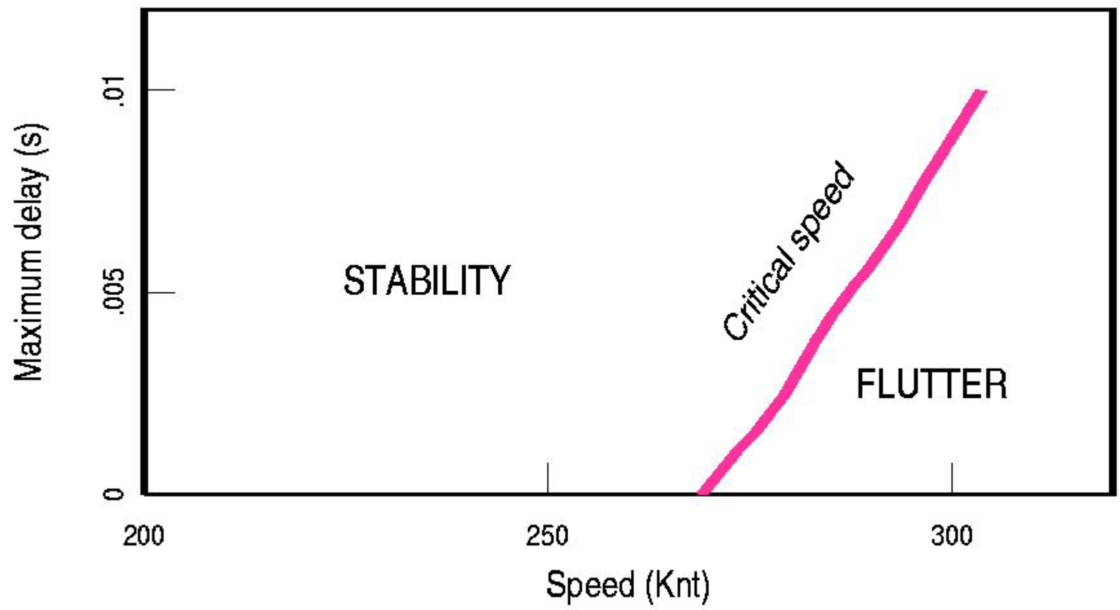


Fig.12 Stability of an aeroservoelastic system with random delay (Poi94b)

5.0 CONCLUSION

Uncertainty appears in many places in aeroelasticity: structural parameters, wing geometry, control systems… Taking into account structural uncertainty during the design process of an aircraft is a mandatory stage in view of flutter certification. Because flutter calculations involve highly nonlinear equations, MCS methods are the more appropriate ones for introducing random uncertain parameters in the model and evaluating their impacts on the stability of the coupled system aircraft/aerodynamics. Even if the complete MCS method will give the more accurate results, nevertheless reduced methods are required today in order to address the computational cost due to the high dimension of the finite element models used by manufacturers. A feasible and effective approach has been presented, based on the construction of an unique representative basis on which the flutter equation is projected. Moreover, in order to obtain a more accurate representation of the uncertainty propagation in the various structural matrices, the use of Hermite polynomial chaos has been suggested, instead of the classical first order perturbation technique. A simple example has shown the benefit of this approach, together with some of its difficulties. As far as this study in concerned, it appears that a second order polynomial chaos expansion is sufficient to obtain satisfactory results. The second application on a realistic case has shown the feasibility of the polynomial chaos approach for modelling random structural matrices in real life engineering structures, but has also highlighted the importance of using accurate coefficients in the expansion. Since those coefficients are obtained numerically (either using adapted numerical integration methods when few uncertain parameters are involved or using MCS as soon as more than four parameters are introduced) for nonlinear problems, particular efforts are still needed to improve their accuracy and limit their computational cost. Finally the question arises whether such simplified approaches should be used, at least for middle size (several dozens of thousand degrees of freedom) models are involved, since MCS methods appear to converge rapidly (at least for the flutter problem) and since the computational power of computers increases continuously. Introducing purely aerodynamic uncertainties is a more delicate process. Here we have shown that a Monte Carlo procedure coupled with a non linear Euler solver could permit to take into account geometric uncertainties for a 2 DOF airfoil. The cost for such an approach is however expensive and it seems unlikely that it could be extended do 3D structures. Finally we have shown the possibility of taking into account uncertain time delays in controlled systems.

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Paper No. 38

Discusser's Name: L. Fatini

Question: (1) If the aerodynamic problem is solved in 2-D, how can the structural problem be solved considering the torsion of the wing? (2) Do you have any experimental results? (3) What kind of materials did you use for the models?
Author's Reply: (1) Not solved. (2) No. (3) Metallic.